Mean and Variance Value-Added Indicators with Multilevel Shrinkage: Application to a Multi-District Statewide Value-Added System

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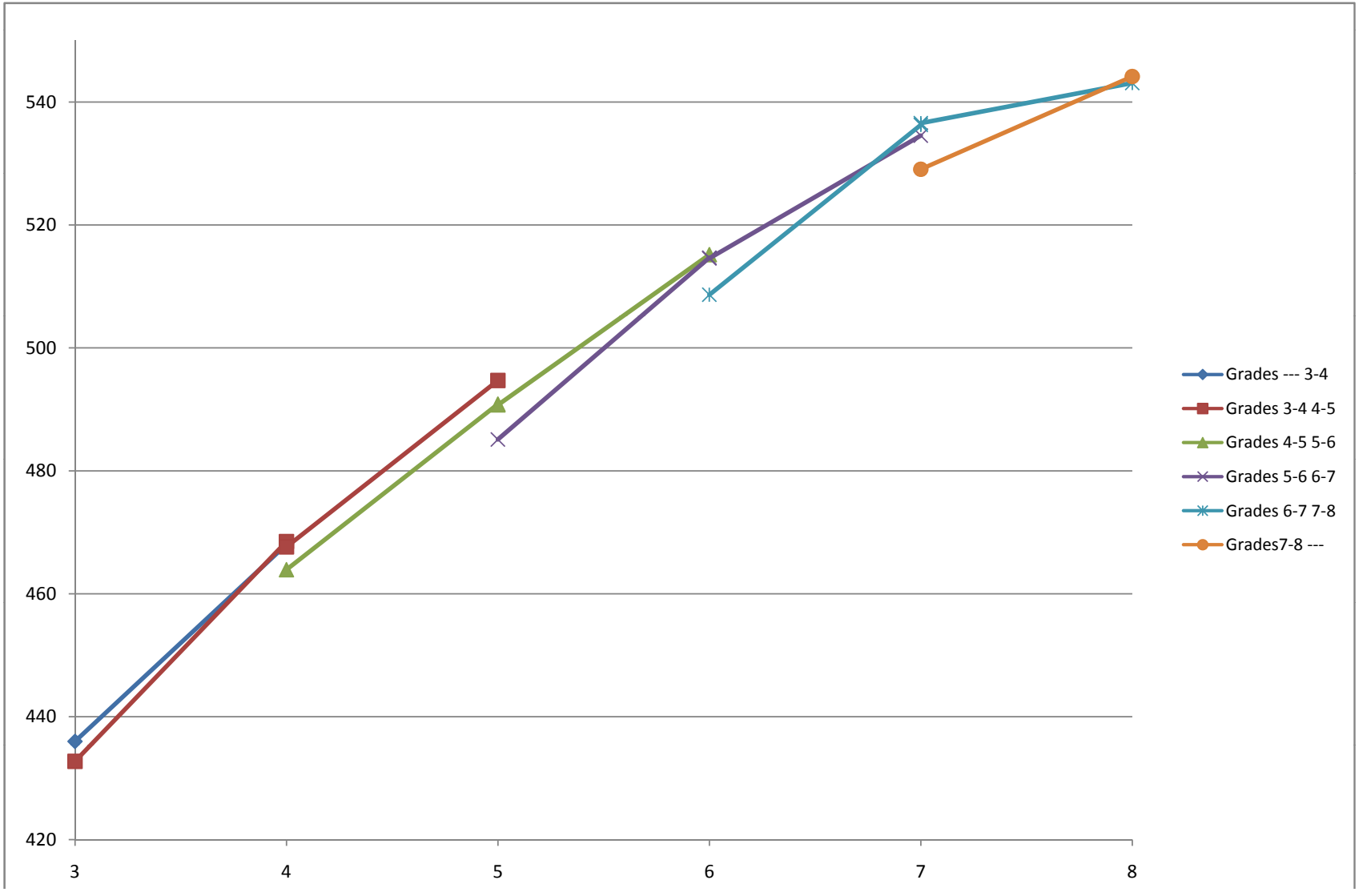
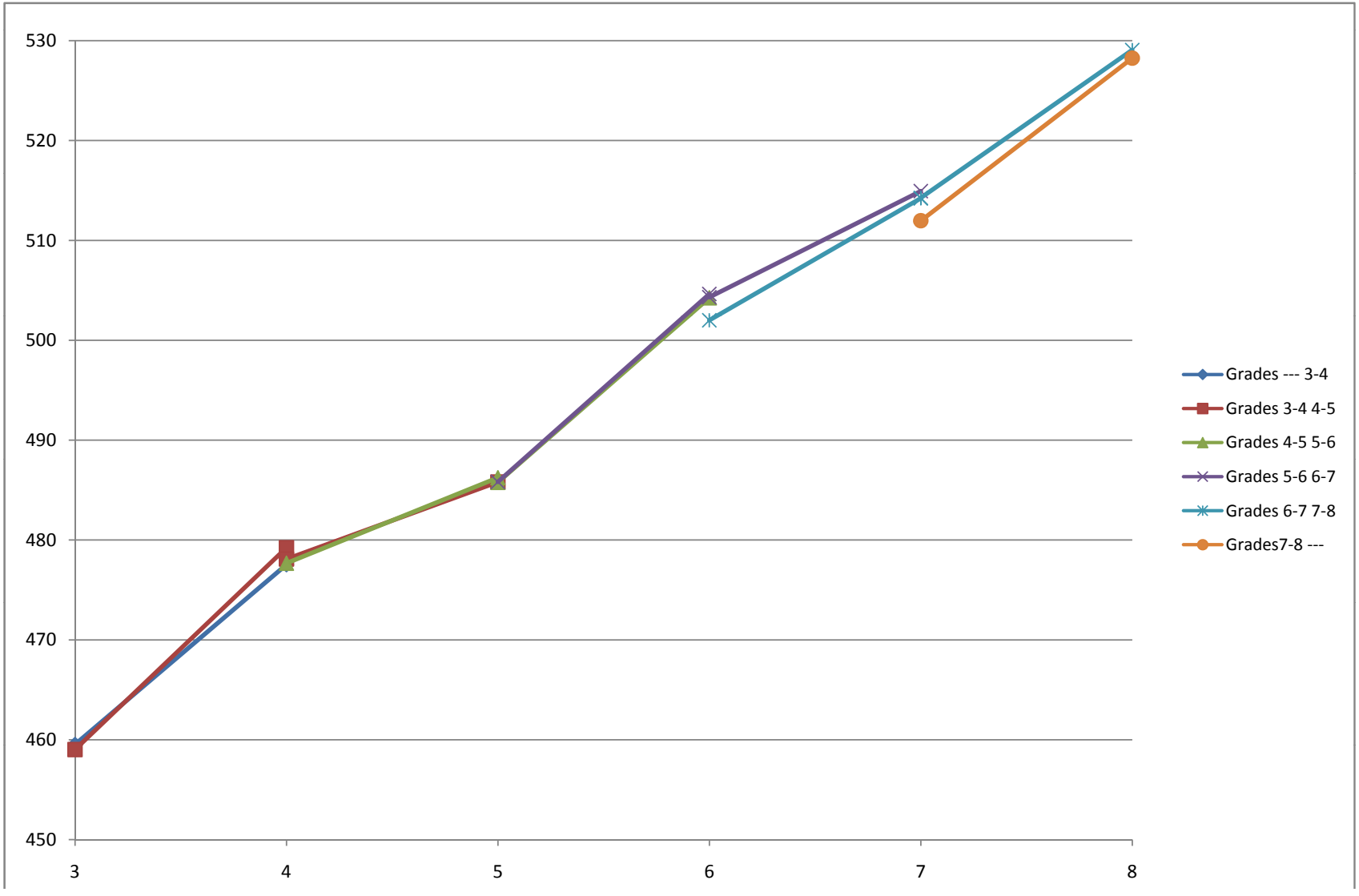
VARC

Value-Added Research Center

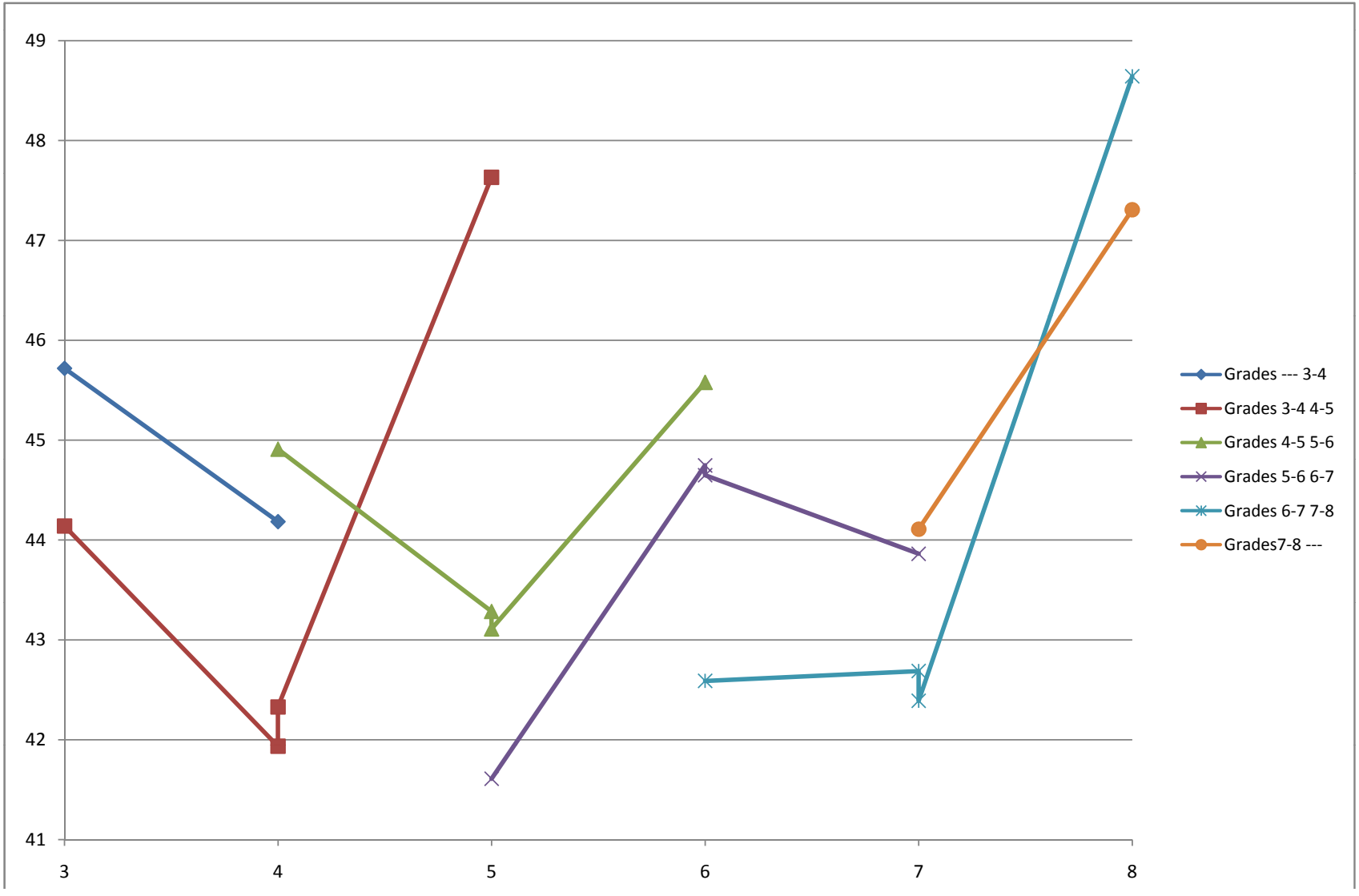
Growth in Average Mathematics Scale Score



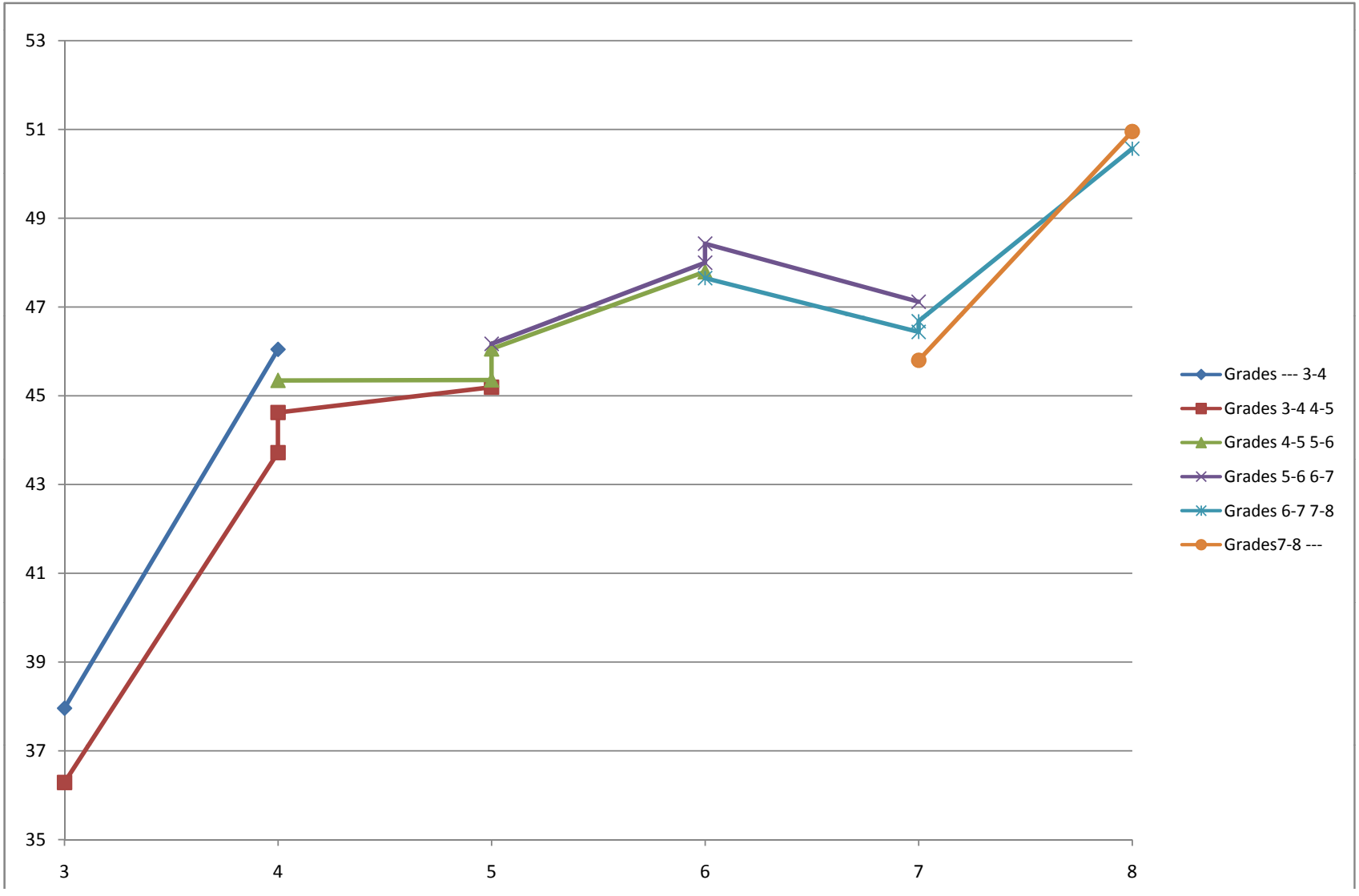
Growth in Average Reading Scale Score



Standard Deviations of Mathematics Scores across Grades



Standard Deviations of Reading Scores across Grades



Comparison of Average Gain in Achievement in Growth Years 1 (05 - 06) and 2 (06 - 07)

Table. Comparison of Average Gain in Achievement in Growth Years 1 (2005 - 2006) and 2 (2006 - 2007)

	Mathematics						
Grade	Average Gain Growth Year 1 (WKCE units)	Average Gain Growth Year 2 (WKCE units)	Change in Growth (WKCE units)	VA Tier Unit (SD) Growth Year 1	VA Tier Unit (SD) Growth Year 2	Change in Growth (Growth Year 1 Tier units)	Change in Growth (Growth Year 1 Tier units)
3 to 4	35.77	32.04	-3.72	7.69	8.11	-0.48	-0.46
4 to 5	26.95	27.08	0.13	7.73	8.57	0.02	0.02
5 to 6	29.58	24.45	-5.13	7.95	7.84	-0.65	-0.65
6 to 7	27.64	19.97	-7.66	5.91	5.45	-1.30	-1.41
7 to 8	15.10	6.53	-8.58	6.35	5.66	-1.35	-1.51

	Reading						
Grade	Average Gain Growth Year 1 (WKCE units)	Average Gain Growth Year 2 (WKCE units)	Change in Growth (WKCE units)	VA Tier Unit (SD) Growth Year 1	VA Tier Unit (SD) Growth Year 2	Change in Growth (Growth Year 1 Tier units)	Change in Growth (Growth Year 2 Tier units)
3 to 4	20.20	18.00	-2.20	6.10	6.49	-0.36	-0.34
4 to 5	8.56	7.72	-0.84	5.98	6.00	-0.14	-0.14
5 to 6	18.82	18.48	-0.33	6.45	6.50	-0.05	-0.05
6 to 7	12.21	10.62	-1.59	5.16	4.98	-0.31	-0.32
7 to 8	16.29	14.81	-1.48	5.24	4.87	-0.28	-0.30

Note: As a rough rule of thumb, year-to-year changes in average gain that exceed 0.5 tier (VA standard deviation) units are indicative of possible test form effects and thus may not represent genuine changes in average state productivity. Grades in which average gain exceeds this threshold are shaded in the tables.

T2 Value-Added Model

$$(1) \qquad Y_{2iklt} = \xi + \lambda_t Y_{1iklt-1} + \beta' X_{iklt} + \pi_t + \sum_k \sum_l \eta_{klt} S_{iklt} + \varepsilon_{iklt}$$

Posttest Y_2 = Post-on-Pre Link λ * Pretest Y_1 + Student Characteristics β

+ Statewide Productivity π + School & District Effect η + Unknown Student Characteristics ε

T2 Value-Added Model

1. School/district value-added productivity effects η_{klt}.
2. State-wide value-added productivity effects π_t .
3. Two years of longitudinal attainment data for each student .
4. A posttest on pretest parameter λ. This parameter allows for situations where the variances of the posttest and pretest variables may be atypical.
5. Control for measurement in prior achievement $Y_{1iklt-1}$.
6. Demographic variables X_{iklt} to capture differences across students in achievement growth

School and Districts Value-Added Effects

$$\eta_{klt} = \alpha_{klt} + \delta_{lt} \qquad \omega^2_{\alpha kl} = \omega^2_{Skl} + \frac{\omega^2_{Ckl}}{J_{kl}}$$

$$\alpha_{klt} = \nu_{Sklt} + \bar{\nu}_{C.klt}$$

$$\bar{\eta}_{\cdot lt} = \bar{\alpha}_{.lt} + \delta_{lt} \qquad \bar{\omega}^2_{\alpha l} = \omega^2_{Sl} + \frac{\omega^2_{Cl}}{\bar{J}_{\cdot l}}$$

δ_l	District productivity component
ν_{Skl}	School productivity component
ν_{Cjkl}	Classroom productivity component
$\bar{\nu}_{C.kl}$	Average classroom productivity
$\bar{\eta}_{.l}$	Average district productivity
$\bar{\alpha}_{.l}$	Average of within-district school productivity
ω^2_{Dl}	Variance of δ_l
ω^2_{Skl}	Variance of ν_{Skl}
ω^2_{Ckl}	Variance of ν_{Cjkl}
$\omega^2_{\alpha kl}$	Variance of α_{kl}
J_{kl}	The number of classrooms (at a given grade) in school k in district l

Shrinkage Estimate of η_{kl}

$$\hat{\eta}_l^* = \Omega_l(\Omega_l + \Sigma_l)^{-1}\hat{\eta}_l$$

$$\Omega_l = \omega_D^2 J_l + \omega_{\alpha_l}^2 I_l \quad (2)$$

$$J_l = \mathbf{1}_l \mathbf{1}_l{}'$$

For a 2-school district

$$\Omega_l = \begin{pmatrix} \omega_D^2 + \omega_{\alpha_l}^2 & \omega_D^2 \\ \omega_D^2 & \omega_D^2 + \omega_{\alpha_l}^2 \end{pmatrix}$$

$\hat{\eta}_l^*$	Shrinkage estimate of vector of η_{kl} for district l.
$\hat{\eta}_l$	Estimate of vector of η_{kl} for district l.
Ω_l	Covariance matrix of (true) η_{kl} for district l.
Σ_l	Covariance matrix of estimation error for $\hat{\eta}_{kl}$ for district l.
ω_D^2	Variance of district effect δ_l.
$\omega_{\alpha_l}^2$	Variance of school effect α_{kl} in district l.

Details on ω_δ^2 and $\omega_{\alpha_l}^2$

$$
\begin{aligned}
\omega_T^2 &= \omega_D^2 + \overline{\omega}_{\alpha.}^2 + \overline{\sigma}_{..}^2 \\
\omega_{\alpha_l}^2 &= \frac{\Delta_l^2 - \left(\overline{\sigma}_{.l}^2 - \sum_k (\frac{n_{kl}}{N_l})^2 \sigma_{kl}^2\right)}{1 - \sum_k (\frac{n_{kl}}{N_l})^2} \text{ for } K_l \geq 2 \\
\Delta_l^2 &= \sum_k \frac{n_{kl}}{N_l} (\hat{\eta}_{kl} - \bar{\hat{\eta}}_{.l})^2 \qquad (3) \\
\omega_D^2 &= \omega_{T(2)}^2 - \overline{\omega}_{\alpha.(2)}^2 - \overline{\sigma}_{..(2)}^2 \\
\overline{\omega}_{\alpha.(1)}^2 &= \omega_{T(1)}^2 - \omega_D^2 - \overline{\sigma}_{..(1)}^2
\end{aligned}
$$

Coefficients on Prior Achievement (Corrected for Measurement Error) 05-06

Nov 2005 - Nov 2006						
Grades	Variable	Math		Reading		N
		Estimate	Std Err	Estimate	Std Err	
3 to 4	Post/Pre Test SD Ratio	0.95		1.20		53891
	Pre Test in SD Units	0.86	0.003	0.90	0.003	
	Pre Test	0.81	0.003	1.08	0.004	
4 to 5	Post/Pre Test SD Ratio	0.96		1.00		55872
	Pre Test in SD Units	0.85	0.003	0.87	0.003	
	Pre Test	0.82	0.003	0.87	0.003	
5 to 6	Post/Pre Test SD Ratio	1.08		1.04		56088
	Pre Test in SD Units	0.86	0.003	0.84	0.003	
	Pre Test	0.92	0.003	0.87	0.003	
6 to 7	Post/Pre Test SD Ratio	1.00		0.97		58955
	Pre Test in SD Units	0.89	0.003	0.90	0.003	
	Pre Test	0.89	0.003	0.88	0.003	
7 to 8	Post/Pre Test SD Ratio	1.07		1.11		60768
	Pre Test in SD Units	0.90	0.003	0.90	0.003	
	Pre Test	0.97	0.003	1.00	0.003	

Coefficients on Prior Achievement (Corrected for Measurement Error) 06-07

Nov 2005 - Nov 2006						
Grades	Variable	Math		Reading		N
		Estimate	Std Err	Estimate	Std Err	
3 to 4	Post/Pre Test SD Ratio	0.95		1.20		53891
	Pre Test in SD Units	0.86	0.003	0.90	0.003	
	Pre Test	0.81	0.003	1.08	0.004	
4 to 5	Post/Pre Test SD Ratio	0.96		1.00		55872
	Pre Test in SD Units	0.85	0.003	0.87	0.003	
	Pre Test	0.82	0.003	0.87	0.003	
5 to 6	Post/Pre Test SD Ratio	1.08		1.04		56088
	Pre Test in SD Units	0.86	0.003	0.84	0.003	
	Pre Test	0.92	0.003	0.87	0.003	
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	Pre Test in SD Units	0.89	0.003	0.90	0.003	
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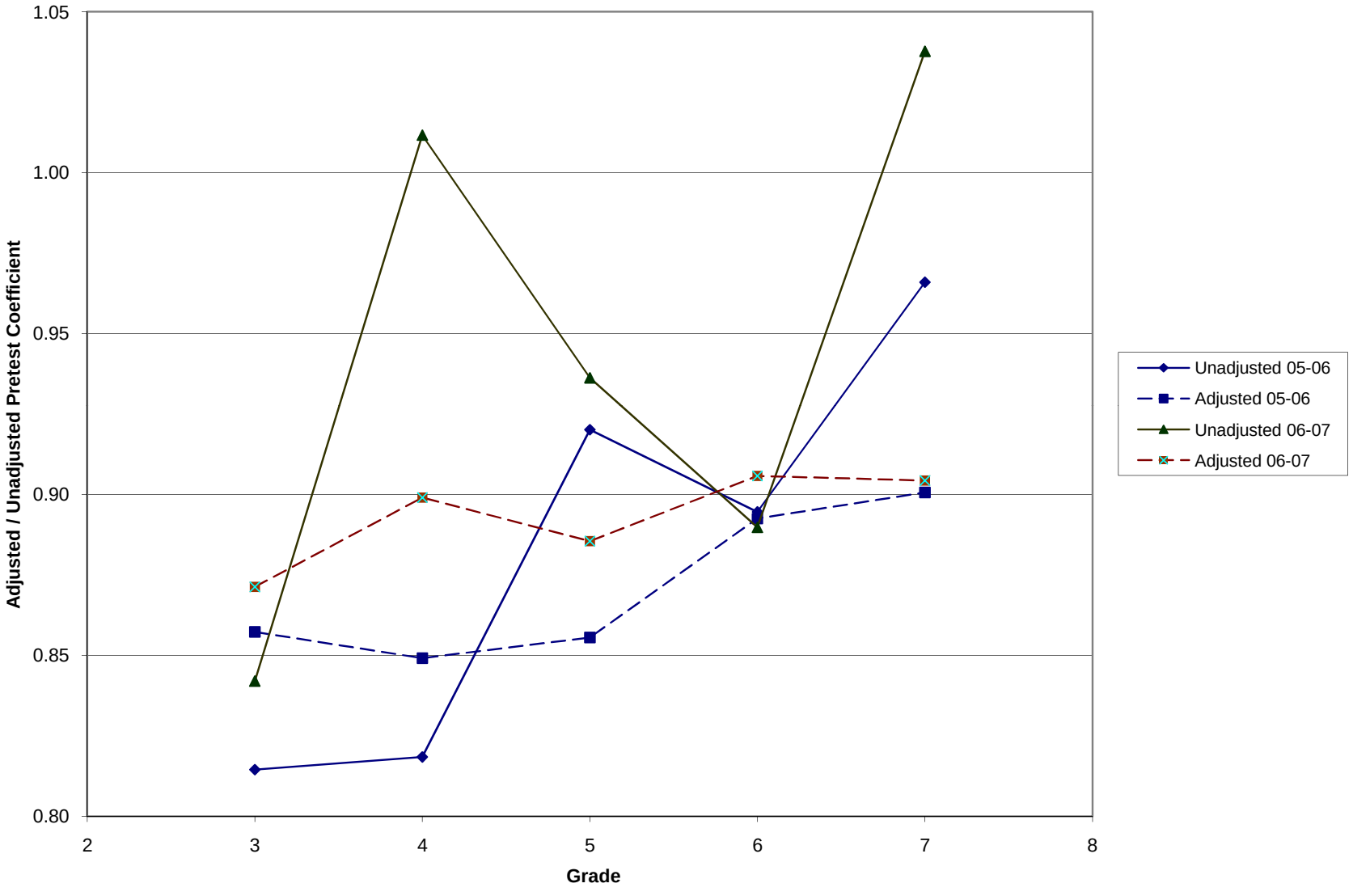
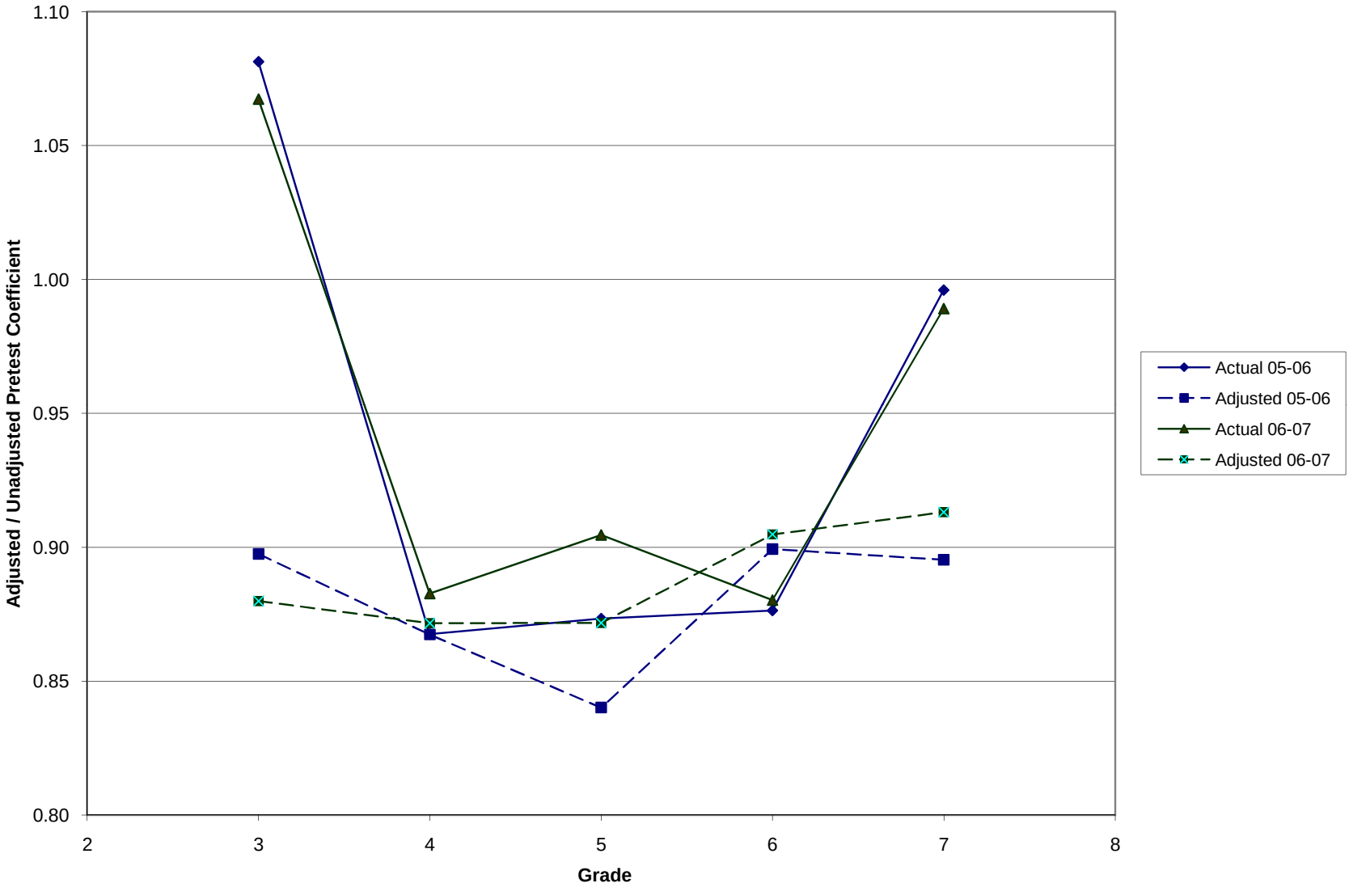
	Pre Test in SD Units	0.90	0.003	0.90	0.003	
	Pre Test	0.97	0.003	1.00	0.003	

Pretest Coefficient on Mathematics



Pretest Coefficient on Reading



So

1. Does average productivity (as represented by the mean parameter $\bar{\eta}_{\cdot lt}$) differ across districts?
2. Does the consistency of school productivity (as captured by the variance parameter $\bar{\omega}^2_{\alpha l}$) differ across districts?
3. Do districts differ systematically in their capacity to create high and low-performing schools? (Is $\omega^2_D > 0$?)

Magnitude of Value-Added Productivity Parameters in Mathematics

	Mathematics									
	Growth Year 1 (2005 - 2006)					Growth Year 2 (2006 - 2007)				
	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8
Standard Deviation of VA Productivity (Noise Corrected)										
Wisconsin	6.79	6.92	7.20	5.41	5.86	7.10	7.65	7.07	4.83	5.06
Madison	3.56	5.60	5.85	5.37	3.05	8.16	3.06	5.75	2.93	2.59
Milwaukee	11.39	8.53	10.22	6.38	6.53	11.34	9.54	8.53	6.57	9.40
Low to High VA Productivity										
Wisconsin	27.14	27.69	28.81	21.64	23.45	28.42	30.61	28.29	19.32	20.25
Madison	14.24	22.41	23.39	21.48	12.21	32.66	12.26	23.02	11.71	10.34
Milwaukee	45.55	34.12	40.88	25.53	26.12	45.35	38.16	34.13	26.30	37.58
Basic to Proficient Gap	15.00	17.00	18.00	21.00	24.00	15.00	17.00	18.00	21.00	24.00
Basic to Advanced Gap	60.00	63.00	60.00	68.00	75.00	60.00	63.00	60.00	68.00	75.00
One Year of Growth	33.91	27.02	27.02	23.81	10.82	33.91	27.02	27.02	23.81	10.82
Student Level Effect Size (Standard Deviation of Post Achievement)	41.93	43.28	44.75	42.69	47.31	44.18	47.63	45.58	43.86	48.64

Magnitude of Value-Added Productivity Parameters in Reading

	Reading									
	Growth Year 1 (2005 - 2006)					Growth Year 2 (2006 - 2007)				
	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8
Standard Deviation of VA Productivity (Noise Corrected)										
Wisconsin	4.73	4.71	5.24	4.33	4.37	5.18	4.72	5.35	3.96	3.96
Madison	4.71	3.45	5.33	5.75	5.63	4.92	4.91	2.83	2.88	4.24
Milwaukee	7.69	8.23	8.63	6.09	6.43	8.28	8.16	8.21	6.08	5.58
Low to High VA Productivity										
Wisconsin	18.92	18.86	20.97	17.32	17.49	20.72	18.86	21.39	15.83	15.85
Madison	18.84	13.79	21.33	23.00	22.51	19.69	19.66	11.30	11.51	16.95
Milwaukee	30.75	32.91	34.53	24.38	25.72	33.12	32.62	32.82	24.31	22.30
Basic to Proficient Gap	36.00	44.00	43.00	39.00	33.00	36.00	44.00	43.00	39.00	33.00
Basic to Advanced Gap	72.00	93.00	96.00	96.00	89.00	72.00	93.00	96.00	96.00	89.00
One Year of Growth	19.10	8.14	18.65	11.42	15.55	19.10	8.14	18.65	11.42	15.55
Student Level Effect Size (Standard Deviation of Post Achievement)	43.72	45.35	48.00	46.44	50.95	46.04	45.19	47.79	47.12	50.57

Estimates of the Reliability and Variance of District and Within-District School Effects in Wisconsin

	Mathematics									
	Growth Year 1 (2005 - 2006)					Growth Year 2 (2006 - 2007)				
	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8
Variance of district VA effects	9.50	16.20	20.39	0.0	9.68	2.54	18.75	17.09	3.24	0.25
Average variance of VA within-district school effects	36.54	31.72	31.51	29.28	24.70	47.94	39.79	32.94	20.09	25.39
Variance of total school/district VA effects	46.05	47.93	51.89	29.28	34.38	50.48	58.55	50.03	23.33	25.64
Average variance of statistical estimation error (noise)	13.07	11.75	11.31	5.69	5.94	15.22	14.89	11.47	6.34	6.44
Total variance, including statistical estimation error	59.11	59.68	63.20	34.97	40.32	65.70	73.44	61.50	29.67	32.08
Average reliability of total school/district VA effects	0.78	0.80	0.82	0.84	0.85	0.77	0.80	0.81	0.79	0.80
	Reading									
	Growth Year 1 (2005 - 2006)					Growth Year 2 (2006 - 2007)				
	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8
Variance of district VA effects	4.92	7.35	10.19	8.67	1.19	5.98	7.11	13.60	9.12	0.0
Average variance of VA within-district school effects	17.46	14.89	17.29	10.07	17.91	20.84	15.13	15.00	6.56	15.70
Variance of total school/district VA effects	22.37	22.23	27.48	18.75	19.11	26.82	22.24	28.60	15.67	15.70
Average variance of statistical estimation error (noise)	14.88	13.55	14.10	7.86	8.40	15.28	13.81	13.60	9.17	7.99
Total variance, including statistical estimation error	37.26	35.78	41.58	26.61	27.51	42.10	36.05	42.20	24.84	23.69
Average reliability of total school/district VA effects	0.60	0.62	0.66	0.70	0.69	0.64	0.62	0.68	0.63	0.66

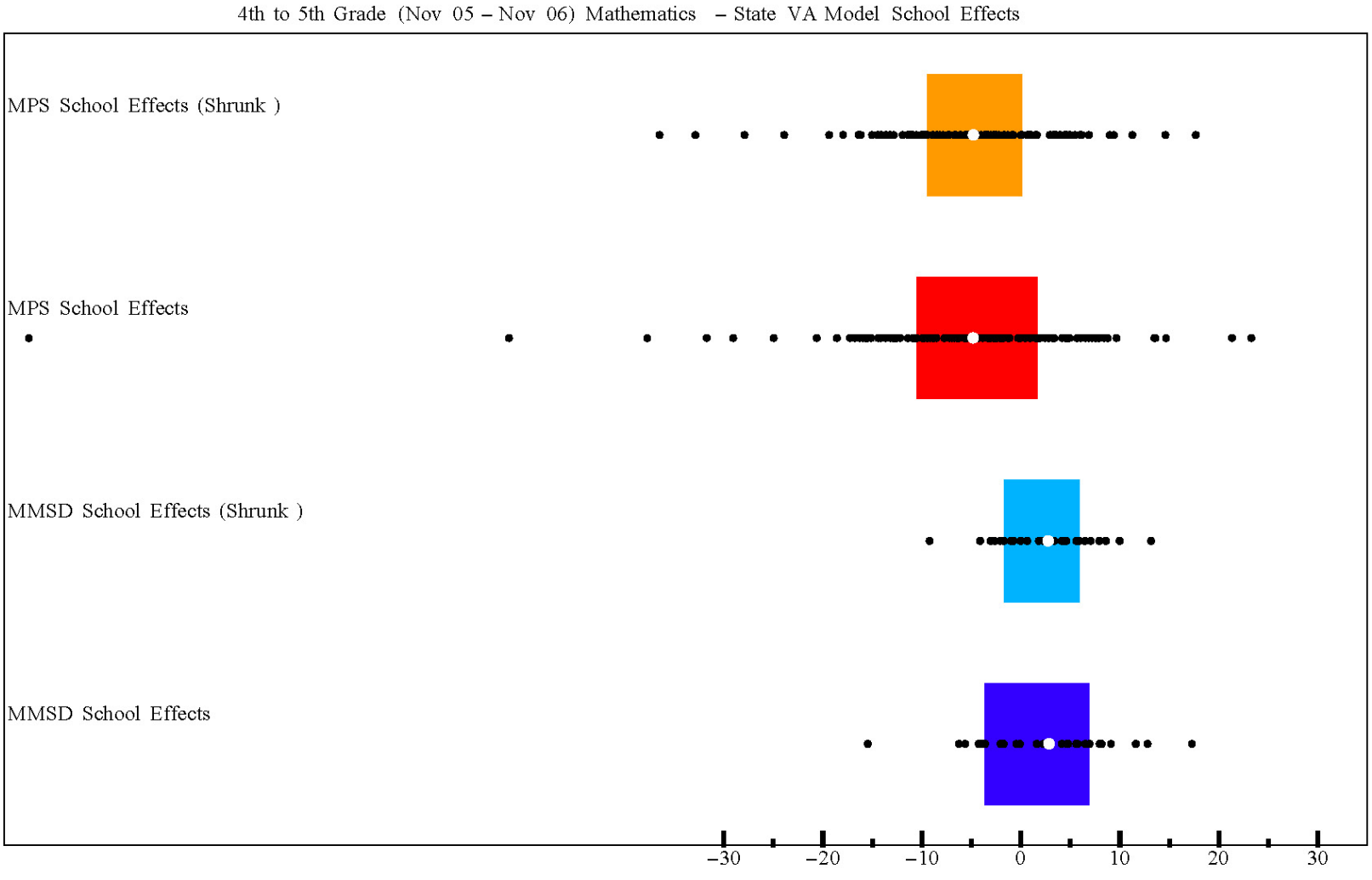
Estimates of the Reliability and Variance of District and Within-District School Effects in Milwaukee

	Milwaukee									
	Math 05-06					Math 06-07				
	3	4	5	6	7	3	4	5	6	7
Variance of VA within-district school effects	129.67	72.76	104.45	40.75	42.64	128.52	91.02	72.82	43.23	88.27
Average variance of statistical estimation error (noise)	15.16	13.04	13.61	8.17	9.98	17.28	17.54	14.45	9.82	10.92
Total variance, including statistical estimation error	144.83	85.80	118.07	48.92	52.62	145.80	108.56	87.27	53.04	99.19
Reliability	0.90	0.85	0.88	0.83	0.81	0.88	0.84	0.83	0.81	0.89
	Read 05-06					Read 06-07				
	3	4	5	6	7	3	4	5	6	7
Variance of VA within-district school effects	59.08	67.69	74.52	37.14	41.35	68.58	66.51	67.33	36.95	31.09
Average variance of statistical estimation error (noise)	17.26	15.04	16.98	11.29	14.12	17.35	16.27	17.14	14.19	13.54
Total variance, including statistical estimation error	76.35	82.73	91.50	48.43	55.47	85.92	82.78	84.47	51.14	44.63
Reliability	0.77	0.82	0.81	0.77	0.75	0.80	0.80	0.80	0.72	0.70

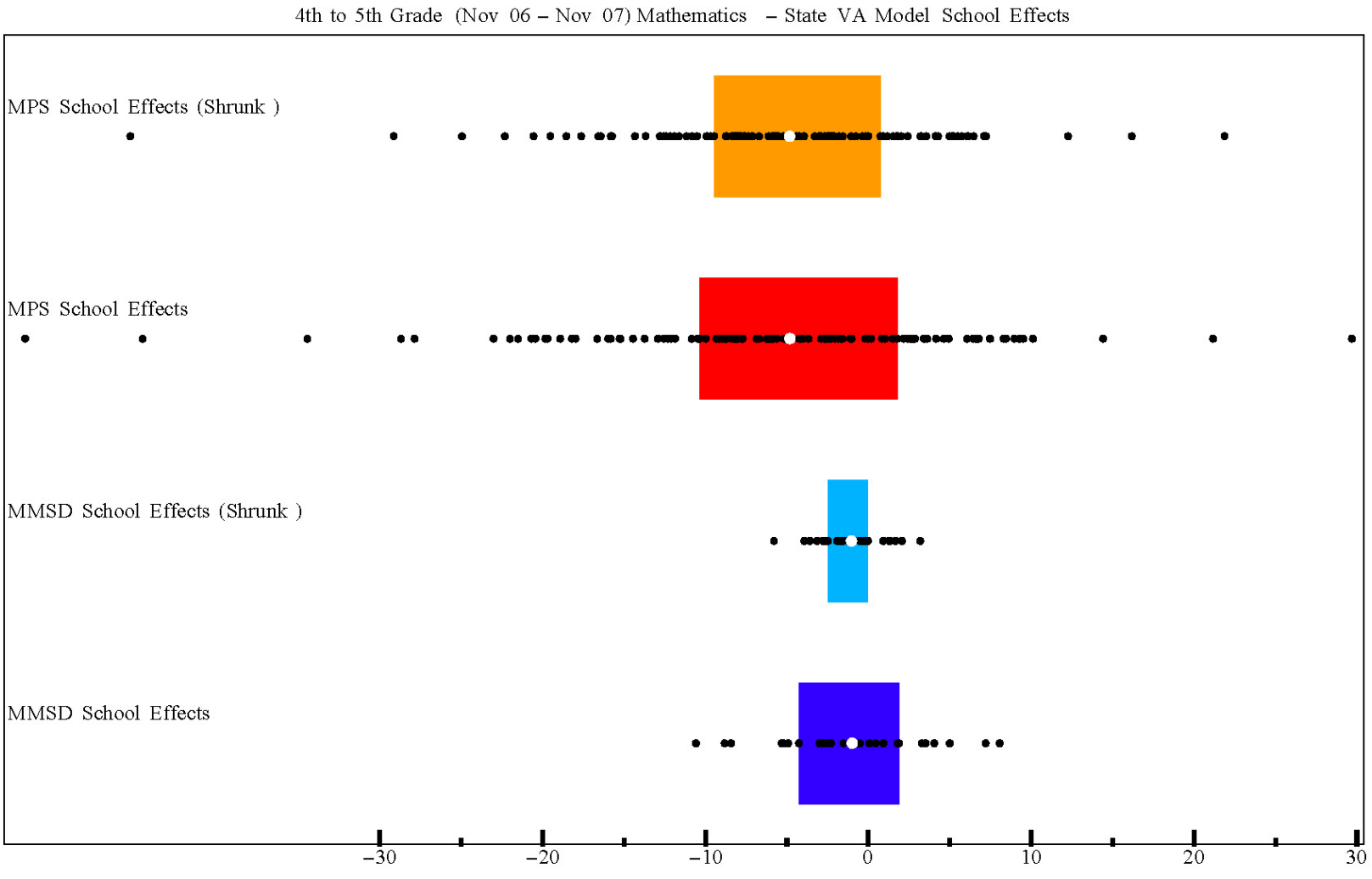
Estimates of the Reliability and Variance of District and Within-District School Effects in Madison

	Madison									
	Math 05-06					Math 06-07				
	3	4	5	6	7	3	4	5	6	7
Variance of VA within-district school effects	12.67	31.39	34.18	28.82	9.31	66.66	9.39	33.12	8.58	6.69
Average variance of statistical estimation error (noise)	10.64	10.22	10.39	3.55	4.19	12.44	12.10	10.08	3.52	4.43
Total variance, including statistical estimation error	23.31	41.61	44.58	32.38	13.50	79.09	21.50	43.20	12.10	11.11
Reliability	0.54	0.75	0.77	0.89	0.69	0.84	0.44	0.77	0.71	0.60
	Read 05-06					Read 06-07				
	3	4	5	6	7	3	4	5	6	7
Variance of VA within-district school effects	22.18	11.89	28.43	33.06	31.66	24.22	24.15	7.98	8.28	17.95
Average variance of statistical estimation error (noise)	12.11	11.78	12.97	4.91	5.93	12.48	11.23	11.96	5.09	5.49
Total variance, including statistical estimation error	34.29	23.67	41.40	37.97	37.58	36.70	35.38	19.94	13.37	23.44
Reliability	0.65	0.50	0.69	0.87	0.84	0.66	0.68	0.40	0.62	0.77

Value-Added Productivity Estimates for Milwaukee and Madison, Growth Year 1



Value-Added Productivity Estimates for Milwaukee and Madison, Growth Year 2



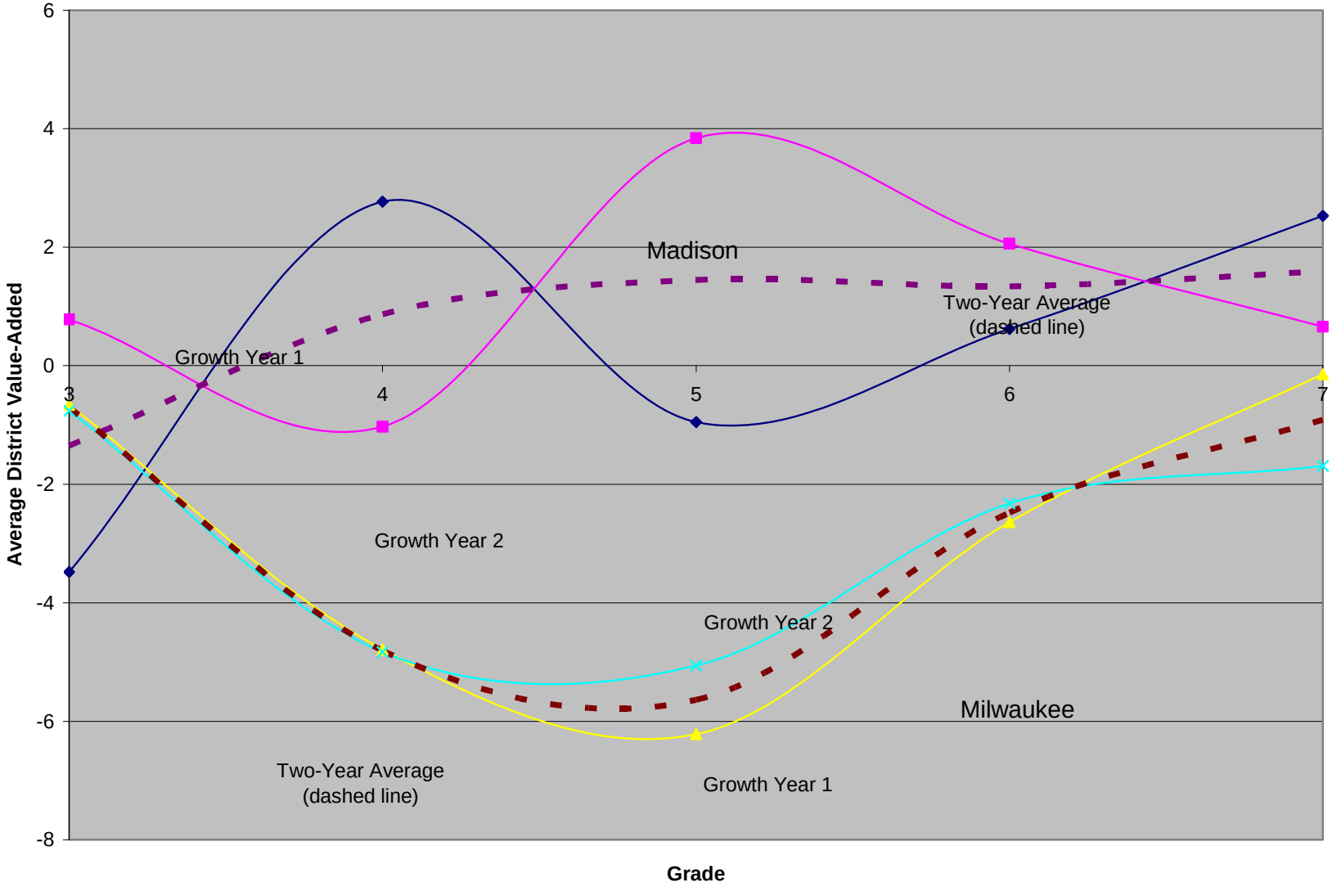
District Math Value-Added Effects: Madison and Milwaukee

		Mathematics					
		Growth Year 1			Growth Year 2		
Grade	District	District Average	Standard Error	District Standard Deviation	District Average	Standard Error	District Standard Deviation
3	Madison	-3.48	0.59	3.56	0.78	0.61	8.16
4	Madison	2.77	0.59	5.6	-1.03	0.64	3.06
5	Madison	-0.95	0.6	5.85	3.84	0.59	5.75
6	Madison	0.62	0.5	5.37	2.06	0.51	2.93
7	Madison	2.53	0.58	3.05	0.66	0.43	2.59
3	Milwaukee	-0.66	0.4	11.39	-0.76	0.41	11.34
4	Milwaukee	-4.79	0.37	8.53	-4.83	0.43	9.54
5	Milwaukee	-6.22	0.39	10.22	-5.06	0.39	8.53
6	Milwaukee	-2.64	0.29	6.38	-2.32	0.36	6.57
7	Milwaukee	-0.14	0.39	6.53	-1.69	0.37	9.4

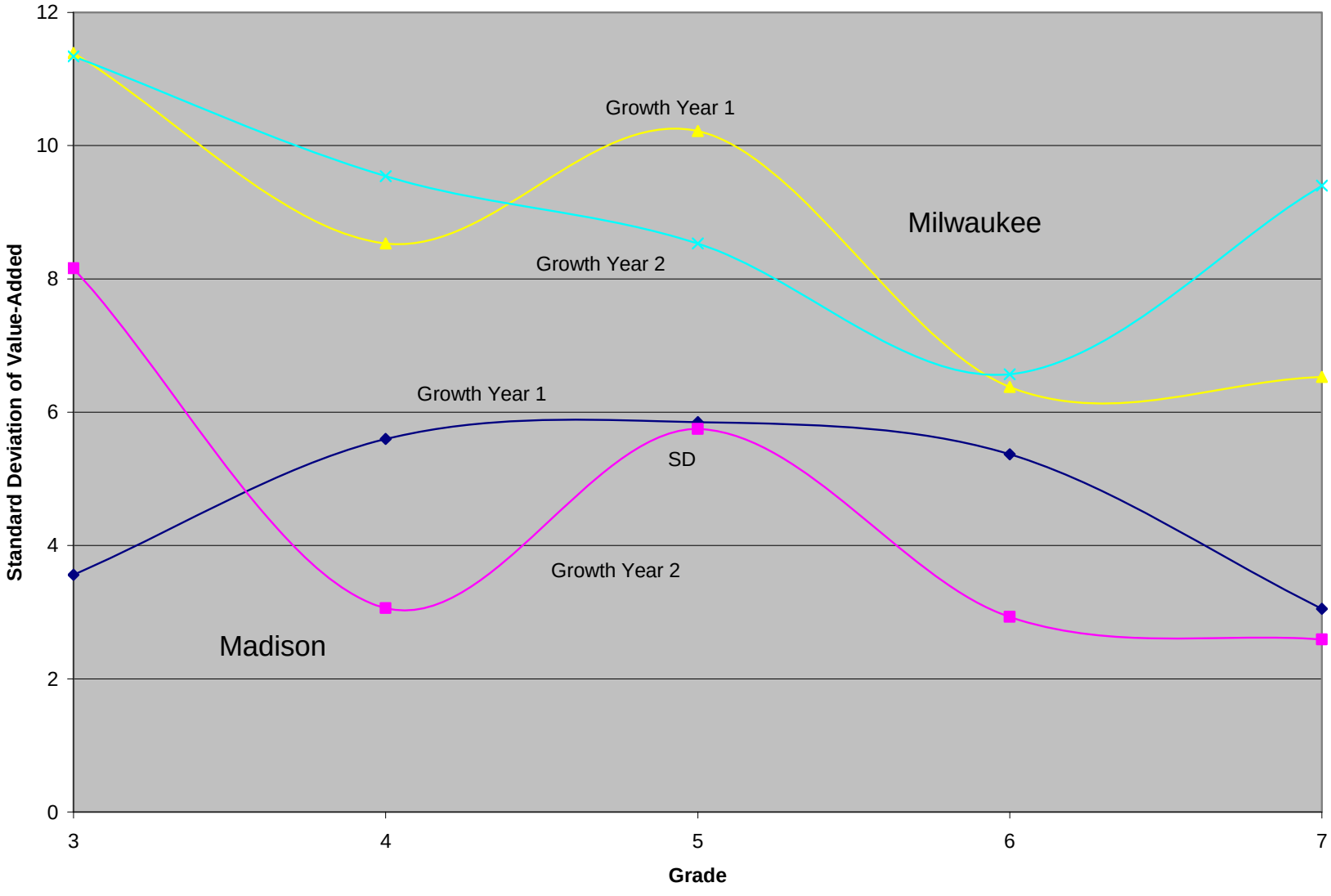
District Reading Value-Added Effects: Madison and Milwaukee

		Reading					
		Growth Year 1			Growth Year 2		
Grade	District	District Average	Standard Error	District Standard Deviation	District Average	Standard Error	District Standard Deviation
3	Madison	0.52	0.61	4.71	-0.49	0.63	4.92
4	Madison	3.36	0.61	3.45	2.59	0.6	4.91
5	Madison	0.9	0.65	5.33	0.82	0.63	2.83
6	Madison	1.01	0.63	5.75	0.91	0.64	2.88
7	Madison	1.35	0.64	5.63	1.32	0.54	4.24
3	Milwaukee	-1.89	0.41	7.69	-5.37	0.41	8.28
4	Milwaukee	-1.99	0.39	8.23	-4.33	0.41	8.16
5	Milwaukee	-4.17	0.42	8.63	-5.13	0.42	8.21
6	Milwaukee	-2.34	0.39	6.09	-2.77	0.43	6.08
7	Milwaukee	1.92	0.42	6.43	0.24	0.34	5.58

Average District Value-Added in Madison and Milwaukee by Grade and Growth Year



Standard Deviation of Value-Added in Madison and Milwaukee by Grade and Growth Year



Low Performing Schools: Madison and Milwaukee

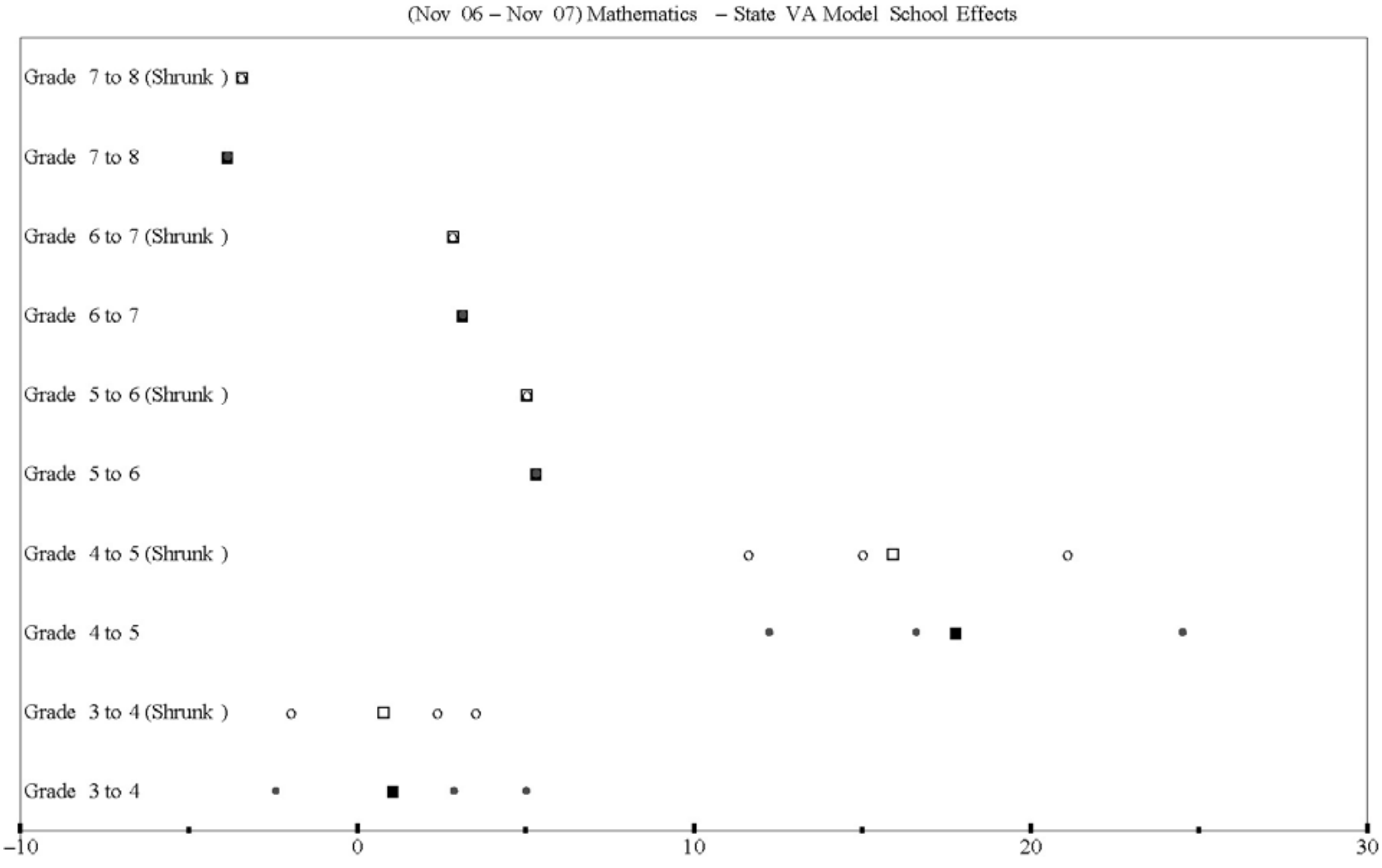
		Mathematics			
		Growth Year 1		Growth Year 2	
Grade	District	Number of Low Performing Schools	Percentage of Low Performing Schools	Number of Low Performing Schools	Percentage of Low Performing Schools
3	Madison	16	59.26	8	29.63
4	Madison	2	7.41	3	11.11
5	Madison	10	37.04	2	7.41
6	Madison	3	27.27	1	9.09
7	Madison	0	0	2	18.18
3	Milwaukee	49	39.52	52	42.62
4	Milwaukee	73	58.4	71	58.2
5	Milwaukee	75	59.52	70	56.91
6	Milwaukee	49	50	59	60.2
7	Milwaukee	26	27.96	48	52.17

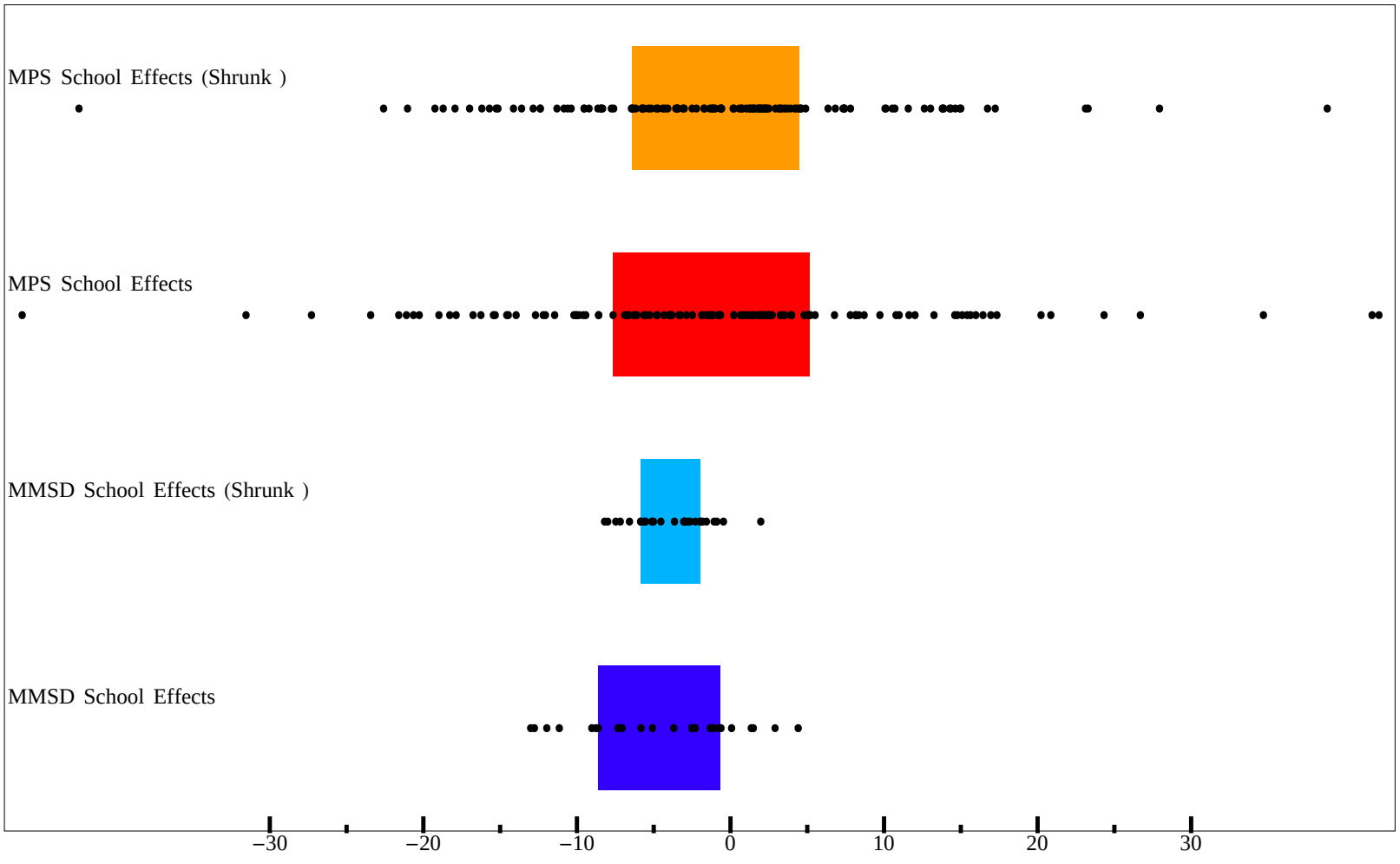
		Reading			
		Growth Year 1		Growth Year 2	
Grade	District	Number of Low Performing Schools	Percentage of Low Performing Schools	Number of Low Performing Schools	Percentage of Low Performing Schools
3	Madison	10	37.04	10	37.04
4	Madison	1	3.7	5	18.52
5	Madison	8	29.63	1	3.7
6	Madison	4	36.36	3	27.27
7	Madison	3	27.27	1	9.09
3	Milwaukee	61	49.19	84	68.85
4	Milwaukee	62	49.6	78	63.93
5	Milwaukee	79	62.7	77	62.6
6	Milwaukee	51	52.04	49	50
7	Milwaukee	21	22.58	27	29.35

District Math Value-Added Effects: A Small District

	Mathematics				Reading			
	Growth Year 1		Growth Year 2		Growth Year 1		Growth Year 2	
Grade	District Average	Standard Error	District Average	Standard Error	District Average	Standard Error	District Average	Standard Error
3	-0.91	1.59	0.79	1.53	-1.36	1.44	-3.2	1.42
4	9.36	1.43	15.93	1.66	0.08	1.34	3.39	1.34
5	5.96	1.51	5.05	1.44	7.74	1.53	5.12	1.47
6	-1.93	1.32	2.85	1.32	1.48	1.39	0.77	1.39
7	-3.91	1.36	-3.42	1.36	2.61	1.45	-2.61	1.45

Value-Added Productivity Estimates for a Small District, Growth Year 2




3rd to 4th Grade (Nov 05 – Nov 06) Mathematics – State VA Model School Effects
MPS School Effects (Shrunk)
MPS School Effects
MMSD School Effects (Shrunk)
MMSD School Effects
−30
−20
−10
0
10
20
30

November 05 - November 06

Number of Schools in District	Number of Districts, Grades 3	4	5	6	7	Avg Number of Std per District, Grades 3	4	5	6	7	Number of Students, Grades 3	4	5	6	7	% of Students, Grades 3	4	5	6	7
1	273	280	297	362	376	48.1	51.0	57.8	83.2	88.7	13124	14286	17179	30105	33338	24.4	25.6	30.6	51.1	54.9
2	48	47	42	32	33	98.3	104.9	118.6	212.8	275.5	4717	4931	4982	6809	9093	8.8	8.8	8.9	11.5	15.0
3	30	29	22	9	8	152.9	157.0	152.0	409.3	596.1	4587	4554	3343	3684	4769	8.5	8.2	6.0	6.2	7.8
4	22	20	16	1		191.9	207.7	209.1	102.0		4222	4153	3345	102		7.8	7.4	6.0	0.2	0.0
5	14	12	11	5	1	232.5	242.8	230.1	282.6	540.0	3255	2913	2531	1413	540	6.0	5.2	4.5	2.4	0.9
6	14	15	13	6		330.9	322.7	331.5	374.3		4633	4841	4309	2246		8.6	8.7	7.7	3.8	0.0
7	2	2	3	2	3	212.5	212.0	208.3	1298.0	1137.3	425	424	625	2596	3412	0.8	0.8	1.1	4.4	5.6
8	1	1	1	1	2	153.0	170.0	176.0	1522.0	1264.0	153	170	176	1522	2528	0.3	0.3	0.3	2.6	4.2
9	3	3	4	1		452.0	465.0	449.0	498.0		1356	1395	1796	498		2.5	2.5	3.2	0.8	0.0
10	2	2	1	1		395.0	372.5	387.0	366.0		790	745	387	366		1.5	1.3	0.7	0.6	0.0
11	1	1	1	1	1	417.0	459.0	398.0	1447.0	1547.0	417	459	398	1447	1547	0.8	0.8	0.7	2.5	2.5
12	4	4	4	1		533.8	560.5	580.3	596.0		2135	2242	2321	596		4.0	4.0	4.1	1.0	0.0
14	2	2	2			585.5	597.0	618.0			1171	1194	1236			2.2	2.1	2.2	0.0	0.0
17	1	1	1	1		805.0	799.0	850.0	873.0		805	799	850	873		1.5	1.4	1.5	1.5	0.0
18	1	1	1			658.0	656.0	669.0			658	656	669			1.2	1.2	1.2	0.0	0.0
21	2	2	2	1		1125.5	1147.5	1137.0	1046.0		2251	2295	2274	1046		4.2	4.1	4.1	1.8	0.0
25	1	1	1			1340.0	1431.0	1438.0			1340	1431	1438			2.5	2.6	2.6	0.0	0.0
27	2	2	2			1374.5	1380.0	1396.5			2749	2760	2793			5.1	4.9	5.0	0.0	0.0
93					1					5541.0					5541	0.0	0.0	0.0	0.0	9.1
98				1					5652.0					5652		0.0	0.0	0.0	9.6	0.0
124	1					5103.0					5103					9.5	0.0	0.0	0.0	0.0
125		1					5624.0					5624				0.0	10.1	0.0	0.0	0.0
126			1					5436.0					5436			0.0	0.0	9.7	0.0	0.0
Total	424	426	425	425	425	14209.3	14959.6	14842.1	14762.2	10989.7	53891	55872	56088	58955	60768	100.0	100.0	100.0	100.0	100.0

November 06 - November 07

Number of Schools in District	Number of Districts (Grades 3)	(4)	(5)	(6)	(7)	Avg Number of Std per District (Grades 3)	(4)	(5)	(6)	(7)	Number of Students (Grades 3)	(4)	(5)	(6)	(7)	% of Students (Grades 3)	(4)	(5)	(6)	(7)
1	279	283	301	364	374	49.4	51.1	59.4	80.5	88.7	13779	14475	17892	29317	33188	24.8	26.1	31.7	51.0	55.4
2	43	42	38	28	30	105.0	103.5	122.9	228.1	248.9	4515	4345	4669	6388	7467	8.1	7.8	8.3	11.1	12.5
3	30	28	21	10	10	165.3	168.5	157.9	420.2	522.6	4960	4717	3316	4202	5226	8.9	8.5	5.9	7.3	8.7
4	20	20	16	1	1	202.9	203.0	201.5	86.0	732.0	4057	4060	3224	86	732	7.3	7.3	5.7	0.1	1.2
5	15	12	11	6	1	235.5	244.3	243.4	286.3	497.0	3533	2931	2677	1718	497	6.4	5.3	4.7	3.0	0.8
6	14	14	13	4		322.1	326.6	331.4	403.3		4509	4573	4308	1613		8.1	8.3	7.6	2.8	0.0
7	2	3	3	2	3	293.0	278.7	288.0	1299.0	1096.3	586	836	864	2598	3289	1.1	1.5	1.5	4.5	5.5
8	1	1	1			131.0	155.0	169.0			131	155	169			0.2	0.3	0.3	0.0	0.0
9	4	3	4	3	1	444.8	437.7	438.5	780.7	1555.0	1779	1313	1754	2342	1555	3.2	2.4	3.1	4.1	2.6
10	1	2	1		1	404.0	430.5	411.0		1066.0	404	861	411		1066	0.7	1.6	0.7	0.0	1.8
11	1	1	1	1	1	391.0	427.0	446.0	1609.0	1558.0	391	427	446	1609	1558	0.7	0.8	0.8	2.8	2.6
12	4	4	4	1		543.5	561.5	566.5	568.0		2174	2246	2266	568		3.9	4.1	4.0	1.0	0.0
14	2	2	2			602.5	607.5	614.5			1205	1215	1229			2.2	2.2	2.2	0.0	0.0
17	1	1	1	1		825	799	800	860		825	799	800	860		1.5	1.4	1.4	1.5	0.0
18	1	1	1			656	679	654			656	679	654			1.2	1.2	1.2	0.0	0.0
21	2	2	2	1		1164	1147	1133	982		2328	2294	2265	982		4.2	4.1	4.0	1.7	0.0
25	1	1	1			1505	1440	1476			1505	1440	1476			2.7	2.6	2.6	0.0	0.0
27	2	2	2			1434	1419	1417			2868	2838	2834			5.2	5.1	5.0	0.0	0.0
92					1					5323					5323	0.0	0.0	0.0	0.0	8.9
98				1					5177					5177		0.0	0.0	0.0	9.0	0.0
122	1	1				5333	5175				5333	5175				9.6	9.3	0.0	0.0	0.0
123			1					5143					5143			0.0	0.0	9.1	0.0	0.0
Total	424	423	424	423	423	14806.9	14653.8	14672.5	12780.1	12687.6	55538	55379	56397	57460	59901	100.0	100.0	100.0	100.0	100.0

Summary Statistics of Tests in Matched Samples

Subject	Grade		Nov 2005 - Nov 2006					Nov 2006 - Nov 2007				
			N	MEAN		STD		N	MEAN		STD	
	Pre	Post		Pre	Post	Pre	Post		Pre	Post	Pre	Post
Math	3	4	53891	432.72	468.49	44.14	41.93	55538	435.97	468.01	45.72	44.18
	4	5	55872	463.90	490.85	44.91	43.28	55379	467.62	494.70	42.33	47.63
	5	6	56088	485.10	514.68	41.61	44.75	56397	490.72	515.17	43.11	45.58
	6	7	58955	508.64	536.28	42.59	42.69	57460	514.56	534.53	44.65	43.86
	7	8	60768	529.06	544.16	44.11	47.31	59901	536.58	543.10	42.39	48.64
Reading	3	4	53891	459.01	479.22	36.28	43.72	55538	459.52	477.52	37.96	46.04
	4	5	55872	477.67	486.23	45.34	45.35	55379	478.08	485.80	44.62	45.19
	5	6	56088	485.81	504.63	46.17	48.00	56397	485.76	504.24	46.05	47.79
	6	7	58955	502.00	514.21	47.65	46.44	57460	504.31	514.93	48.43	47.12
	7	8	60768	511.96	528.25	45.80	50.95	59901	514.25	529.06	46.68	50.57

Details on Shrinkage Estimate of η_{kl}

$$
\begin{aligned}
E(\eta_l - \hat{\eta}_l^*)(\eta_l - \hat{\eta}_l^*)' &= E(\eta_l - \mathbf{w}_l\hat{\eta}_l)(\eta_l - \mathbf{w}_l\hat{\eta}_l)' \\
&= E(\eta_l - \mathbf{w}_l(\eta_l + \varepsilon_l))(\eta_l - \mathbf{w}_l(\eta_l + \varepsilon_l))' \\
&= E((I - \mathbf{w}_l)\eta_l - \mathbf{w}_l\varepsilon_l)((I - \mathbf{w}_l)\eta_l - \mathbf{w}_l\varepsilon_l)' \\
&= E((I - \mathbf{w}_l)\eta_l\eta_l'(I - \mathbf{w}_l)' - \mathbf{w}_l\varepsilon_l\varepsilon_l'\mathbf{w}_l') \\
&= (I - \mathbf{w}_l)\Omega_l(I - \mathbf{w}_l)' + \mathbf{w}_l\Sigma_l\mathbf{w}_l'
\end{aligned} \tag{4}
$$

FOC:

$$
\begin{aligned}
0 &= -2(I - \mathbf{w}_l)\Omega_l + 2\mathbf{w}_l\Sigma_l \\
0 &= -\Omega_l + \mathbf{w}_l(\Omega_l + \Sigma_l) \qquad (5) \\
\mathbf{w}_l &= \Omega_l(\Omega_l + \Sigma_l)^{-1}
\end{aligned}
$$

Estimate of $\bar{\eta}_{.l}$

$$\bar{\eta}_{.l} = w_l' \eta_l$$
$$\hat{\bar{\eta}}_{.l} = w_l' \hat{\eta}_l$$
$$\hat{\bar{\eta}}^*_{.l} = w_l' (\mathbf{w}_l \hat{\eta}_l) = w_l' \hat{\eta}^*_l$$

$\mathbf{w}_{kl}$	(A vector of) Shrinkage weights of vector of $\hat{\eta}_l$ for shrinkage estimator $\hat{\eta}^*_{kl}$.
w_{kl}	(A number) Weight of η_{kl} for weighted average, $\bar{\eta}_{.l}$.
$\mathbf{w}_l$	(A matrix of) Shrinkage weights of a vector of $\hat{\eta}_l$ for shrinkage estimator $\hat{\eta}^*_l$, i.e. $\mathbf{w}_l = \Omega_l(\Omega_l + \Sigma_l)^{-1}$
w_l	(A vector of) Weights of η_l for weighted average, $\bar{\eta}_{.l}$.

Details on the Estimate of $\bar{\eta}_{.l}$

$$
\begin{aligned}
E(\bar{\eta}_l - \hat{\bar{\eta}}_l^*)(\bar{\eta}_l - \hat{\bar{\eta}}_l^*)' &= E(w_l'\eta_l - \kappa_l'\hat{\eta}_l)(w_l'\eta_l - \kappa_l'\hat{\eta}_l)' \\
&= E(w_l'\eta_l - \kappa_l'(\eta_l + \varepsilon_l))(w_l'\eta_l - \kappa_l'(\eta_l + \varepsilon_l))' \\
&= E((w_l' - \kappa_l')\eta_l + \kappa_l'\varepsilon_l)(w_l' - \kappa_l')\eta_l + \kappa_l'\varepsilon_l)' \\
&= E((w_l' - \kappa_l')\eta_l\eta_l'(w_l' - \kappa_l')' + \kappa_l'\varepsilon_l\varepsilon_l'\kappa_l) \\
&= (w_l' - \kappa_l')\Omega_l(w_l' - \kappa_l')' + \kappa_l'\Sigma\kappa_l
\end{aligned} \tag{6}
$$

FOC:

$$
\begin{aligned}
0 &= -2(w_l' - \kappa_l')\Omega_l + 2\kappa_l'\Sigma \\
0 &= -w_l'\Omega_l + \kappa_l'(\Omega_l + \Sigma_l) \\
\kappa_l' &= w_l'\Omega_l(\Omega_l + \Sigma_l)^{-1} \\
\kappa_l' &= w_l'\mathbf{w}_l
\end{aligned} \tag{7}
$$

Hence $\hat{\eta}_l^* = \kappa_l'\hat{\eta}_l = w_l'\mathbf{w}_l\hat{\eta}_l = w_l'\hat{\eta}_l^*$.

Estimates of Slope Coefficients for Demographic Variables, Growth Year 1, Grades 3 to 5

Nov 2005 - Nov 2006									
Grades	Variable	Math			Reading			Mean	N
		Estimate	Std Err	Pr > |t|	Estimate	Std Err	Pr > |t|		
3 to 4	Female	-0.58	0.22	0.01	-2.87	0.23	0.00	0.49	53891
	African American	-4.24	0.54	0.00	-2.07	0.58	0.00	0.10	
	Hispanic	-0.91	0.57	0.11	0.56	0.61	0.36	0.06	
	Asian	-1.79	0.72	0.01	-0.37	0.77	0.63	0.03	
	Native American	-1.27	1.07	0.23	-2.99	1.14	0.01	0.01	
	Free Lunch Indicator	-1.82	0.28	0.00	-2.36	0.30	0.00	0.31	
	English Language Learner	-1.07	0.70	0.13	-2.56	0.75	0.00	0.04	
	Disability Indicator	-8.52	0.36	0.00	-4.28	0.39	0.00	0.11	
4 to5	Female	1.36	0.21	0.00	3.83	0.22	0.00	0.49	55872
	African American	-1.39	0.52	0.01	-3.88	0.55	0.00	0.10	
	Hispanic	0.26	0.55	0.63	-1.62	0.59	0.01	0.07	
	Asian	4.11	0.67	0.00	-1.71	0.72	0.02	0.03	
	Native American	-0.88	1.00	0.38	-3.35	1.07	0.00	0.01	
	Free Lunch Indicator	-2.33	0.27	0.00	-2.79	0.29	0.00	0.32	
	English Language Learner	-2.03	0.64	0.00	0.07	0.69	0.92	0.05	
	Disability Indicator	-7.00	0.33	0.00	-6.53	0.37	0.00	0.12	
5 to 6	Female	-0.34	0.21	0.10	5.51	0.23	0.00	0.49	56088
	African American	-1.99	0.52	0.00	-4.15	0.58	0.00	0.10	
	Hispanic	0.16	0.55	0.77	-1.93	0.61	0.00	0.06	
	Asian	3.85	0.68	0.00	-3.33	0.75	0.00	0.03	
	Native American	-2.60	0.98	0.01	-1.24	1.10	0.26	0.01	
	Free Lunch Indicator	-2.70	0.27	0.00	-3.01	0.30	0.00	0.32	
	English Language Learner	-0.44	0.65	0.49	-2.27	0.72	0.00	0.05	
	Disability Indicator	-6.23	0.34	0.00	-7.60	0.39	0.00	0.12	

Estimates of Slope Coefficients for Demographic Variables, Growth Year 1, Grades 6 to 7

Nov 2005 - Nov 2006									
Grades	Variable	Math			Reading			Mean	N
		Estimate	Std Err	Pr > |t|	Estimate	Std Err	Pr > |t|		
6 to 7	Female	-1.99	0.18	0.00	-2.90	0.21	0.00	0.49	58955
	African American	-3.31	0.44	0.00	-2.77	0.52	0.00	0.10	
	Hispanic	0.01	0.48	0.98	-0.37	0.57	0.51	0.06	
	Asian	-0.10	0.60	0.87	-0.27	0.71	0.70	0.03	
	Native American	-0.53	0.82	0.52	-0.22	0.96	0.82	0.01	
	Free Lunch Indicator	-2.04	0.23	0.00	-1.73	0.28	0.00	0.31	
	English Language Learner	-1.86	0.60	0.00	-0.96	0.70	0.17	0.04	
	Disability Indicator	-4.84	0.30	0.00	-2.44	0.37	0.00	0.13	
7 to 8	Female	-1.64	0.20	0.00	-1.11	0.24	0.00	0.49	60768
	African American	0.60	0.50	0.23	-1.21	0.59	0.04	0.10	
	Hispanic	-1.09	0.53	0.04	1.28	0.63	0.04	0.06	
	Asian	3.28	0.64	0.00	5.46	0.76	0.00	0.03	
	Native American	-1.67	0.90	0.06	-4.00	1.07	0.00	0.02	
	Free Lunch Indicator	-2.11	0.26	0.00	-1.87	0.31	0.00	0.30	
	English Language Learner	1.50	0.68	0.03	4.79	0.81	0.00	0.03	
	Disability Indicator	-4.24	0.34	0.00	-4.41	0.41	0.00	0.13	

Estimates of Slope Coefficients for Demographic Variables, Growth Year 2, Grades 3 to 5

Nov 2006 - Nov 2007									
Grades	Variable	Math			Reading			Mean	N
		Estimate	Std Err	Pr > |t|	Estimate	Std Err	Pr > |t|		
3 to 4	Female	-2.29	0.24	0.00	-1.90	0.24	0.00	0.49	55538
	African American	-2.92	0.58	0.00	-3.58	0.58	0.00	0.10	
	Hispanic	-0.94	0.61	0.12	-0.41	0.61	0.50	0.08	
	Asian	2.49	0.77	0.00	0.00	0.78	1.00	0.03	
	Native American	-1.65	1.13	0.14	-0.35	1.13	0.75	0.01	
	Free Lunch Indicator	-0.70	0.30	0.02	-2.62	0.31	0.00	0.34	
	English Language Learner	-2.50	0.68	0.00	0.02	0.68	0.98	0.06	
	Disability Indicator	-8.04	0.38	0.00	-4.58	0.39	0.00	0.12	
4 to5	Female	-0.75	0.24	0.00	0.53	0.23	0.02	0.49	55379
	African American	-0.04	0.58	0.94	-4.91	0.55	0.00	0.10	
	Hispanic	0.16	0.61	0.79	-1.21	0.59	0.04	0.07	
	Asian	5.26	0.77	0.00	-0.32	0.74	0.67	0.03	
	Native American	-1.21	1.15	0.29	0.63	1.11	0.57	0.01	
	Free Lunch Indicator	-1.38	0.31	0.00	-1.48	0.29	0.00	0.33	
	English Language Learner	-0.70	0.70	0.32	1.67	0.68	0.01	0.06	
	Disability Indicator	-5.44	0.38	0.00	-5.22	0.38	0.00	0.12	
5 to 6	Female	-0.41	0.21	0.05	-1.08	0.23	0.00	0.49	56397
	African American	-4.93	0.52	0.00	-4.86	0.57	0.00	0.10	
	Hispanic	-1.04	0.55	0.06	0.31	0.60	0.60	0.07	
	Asian	1.51	0.67	0.02	-2.07	0.73	0.00	0.04	
	Native American	-2.12	1.01	0.03	-1.56	1.10	0.15	0.01	
	Free Lunch Indicator	-1.48	0.27	0.00	-2.37	0.30	0.00	0.33	
	English Language Learner	0.75	0.63	0.23	-1.06	0.69	0.12	0.06	
	Disability Indicator	-9.48	0.34	0.00	-6.39	0.38	0.00	0.13	

Estimates of Slope Coefficients for Demographic Variables, Growth Year 2, Grades 6 to 7

Nov 2006 - Nov 2007									
Grades	Variable	Math			Reading			Mean	N
		Estimate	Std Err	Pr > |t|	Estimate	Std Err	Pr > |t|		
6 to 7	Female	-1.47	0.19	0.00	-3.47	0.23	0.00	0.49	57460
	African American	-4.30	0.46	0.00	-1.89	0.56	0.00	0.10	
	Hispanic	-1.29	0.49	0.01	0.45	0.59	0.45	0.07	
	Asian	-0.32	0.61	0.60	1.70	0.73	0.02	0.04	
	Native American	-0.32	0.90	0.72	-2.56	1.08	0.02	0.01	
	Free Lunch Indicator	-1.62	0.25	0.00	-2.29	0.30	0.00	0.32	
	English Language Learner	-4.78	0.57	0.00	-0.86	0.69	0.21	0.05	
	Disability Indicator	-5.52	0.32	0.00	-1.90	0.40	0.00	0.12	
7 to 8	Female	-1.98	0.21	0.00	4.86	0.23	0.00	0.49	59901
	African American	-4.84	0.51	0.00	-1.59	0.56	0.00	0.10	
	Hispanic	-1.89	0.55	0.00	-0.28	0.61	0.65	0.06	
	Asian	1.39	0.68	0.04	2.33	0.76	0.00	0.03	
	Native American	-0.45	0.95	0.64	-1.48	1.06	0.16	0.01	
	Free Lunch Indicator	-2.19	0.27	0.00	-1.05	0.30	0.00	0.31	
	English Language Learner	0.31	0.66	0.64	3.28	0.73	0.00	0.04	
	Disability Indicator	-4.56	0.35	0.00	-6.47	0.39	0.00	0.13